Qn 3 Find The Derivative of the Function

3a) $f(x) = (2x^3 - 5x^2 + 4)^5$

$f'(x) = \frac{d}{dx}(2x^3 - 5x^2 + 4)^5$ Using Chain Rule

$= 5(2x^3 - 5x^2 + 4)^4 \cdot \frac{d}{dx}(2x^3 - 5x^2 + 4)$

$= 5(2x^3 - 5x^2 + 4)^4 (6x^2 - 10x + 0)$

$= 5(2x^3 - 5x^2 + 4)^4 \cdot 2(3x^2 - 5x)$

$= 10(2x^3 - 5x^2 + 4)(3x^2 - 5x)$

= [illegible]

3b) $f(t) = \left(\frac{1}{2t+1}\right)^4$

$f'(t) = \frac{d}{dt}\left(\frac{1}{2t+1}\right)^4$ Using Chain Rule

$= (-4)(2t+1)^{-5} \cdot \frac{d}{dt}[2t+1]$

$= -\dfrac{4\left(2 \cdot \frac{d}{dt}(t) + \frac{d}{dt}(1)\right)}{(2t+1)^5}$

$= -\dfrac{4(2(1)+0)}{(2t+1)^5}$

$= -\dfrac{8}{(2t+1)^5}$

3c) $g(x) = e^{x^2-x}$

$g'(x) = \frac{d}{dx}\left(e^{x^2-x}\right)$

$= e^{x^2-x} \cdot \frac{d}{dx}[x^2-x]$

$= e^{x^2-x}\left(\frac{d}{dx}(x^2) - \frac{d}{dx}(x)\right)$

$= e^{x^2-x}(2x-1)$

3d) $f(t) = t\sin \pi t$

$f'(t) = \frac{d}{dt}(t\sin(\pi)t)$

$= 0$ Derivative of a Constant is Zero

3e) $g(\theta) = \cos^2\theta$

$g'(\theta) = \frac{d}{d\theta}(\cos^2\theta)$

$= 2\cos(\theta) \cdot \frac{d}{d\theta}(\cos\theta)$

$= 2\cos(\theta)(-\sin\theta)$

$= -2\cos(\theta)\sin(\theta)$

Pr 4 Find the Limit or show that it does not exist

4 a) $\lim_{t \to \infty} \left(\sqrt{25t^2+2} - 5t\right)$

$$\sqrt{25t^2+2} - 5t = \frac{\left(\sqrt{25t^2+2} - 5t\right)\left(5t + \sqrt{25t^2+2}\right)}{5t + \sqrt{25t^2+2}} = \frac{2}{5t + \sqrt{25t^2+2}}$$

$$\lim_{t \to \infty} \left(\sqrt{25t^2+2} - 5t\right) = \lim_{t \to \infty} \left(\frac{2}{5t + \sqrt{25t^2+2}}\right)$$

$$= 2\left[\frac{1}{\lim_{t \to \infty} 5t + \sqrt{25t^2+2}}\right]$$

$$= \frac{2}{5\lim_{t \to \infty} t + \lim_{t \to \infty} \sqrt{25t^2+2}}$$

Rem $\lim_{t \to \infty} t = \infty$.

$$= \frac{2}{5\infty + \sqrt{25 \lim_{t \to \infty} t^2}} = \frac{2}{5 \cdot \infty + \sqrt{25 \cdot \infty^2}} = 0$$

4b) $\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)$

$$\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)=\lim_{x\to\infty}\left[\frac{ax-bx}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}\right]$$

By product Rule

$$=\lim_{x\to\infty}(ax-bx)(x^2+ax)^{-1/2}\left[\lim_{x\to\infty}\frac{1}{(x^2+ax)^{1/2}\cdot\sqrt{x^2+bx}+1}\right]$$

$$=\lim_{x\to\infty}\frac{(ax-bx)^2}{x^2+ax}\lim_{x\to\infty}\frac{1}{(x^2+ax)^{-1/2}\sqrt{x^2+bx}+1}$$

$$\frac{1}{\sqrt{(-b+a)^2}}\sqrt{\lim_{x\to\infty}\frac{b^2-2ab+a^2}{1+a/x}}\,(-b+a)\lim_{x\to\infty}\frac{1}{(x^2+ax)^{-1/2}\sqrt{x^2+bx}+1}$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{(-b+a)^2}\lim_{x\to\infty}\frac{1}{(x^2+ax)^{1/2}\sqrt{x^2+bx}+1}$$

$$=\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{\sqrt{(-b+a)^2}\lim_{x\to\infty}(x^2+ax)^{-\frac{1}{2}}\sqrt{x^2+bx}+1}$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{\sqrt{(-b+a)^2}}\left[\frac{1}{\sqrt{\lim_{x\to\infty}\frac{x^2+bx}{x^2+ax}}+1}\right]$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{\sqrt{(-b+a)^2}}\left[\frac{1}{\sqrt{\lim_{x\to\infty}\frac{1+\frac{b}{x}}{1+a/x}}+1}\right], \text{ as } x\to\infty,\ \frac{b}{x}=0,\ \frac{a}{x}=0$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{\sqrt{(-b+a)^2}}\left[\frac{1}{\sqrt{\frac{1}{1}}+1}\right]$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{\sqrt{(-b+a)^2}}\cdot\left[\frac{1}{2}\right]$$

$$\frac{(-b+a)\sqrt{b^2-2ab+a^2}}{2\sqrt{(-b+a)^2}}=\frac{1}{2}(-b+a)$$

c) $\lim_{x \to \infty} (x^2 + 2x^7)$

$\lim_{x \to \infty} (x^2 + 2x^7) = \lim_{x \to \infty} 2x^7$, By Taking The limit of the highest power of x

$\lim_{x \to \infty} (2x^7) = 2 \lim_{x \to \infty} (x^7)$

$= 2 \left[\lim_{x \to \infty} x \right]^7$

$\lim_{x \to \infty} x = \infty$

$2 \left[\lim_{x \to \infty} x \right]^7 = 2 \cdot \infty^7 = \infty$

$\therefore \lim_{x \to \infty} [x^2 + 2x^7) = \infty$

$t \to \infty$

4 d) $\lim\limits_{t \to \infty} \dfrac{t+3}{\sqrt{2t^2-1}}$

$= \lim\limits_{t \to \infty} \dfrac{t+3}{\sqrt{2t^2-1}} = \lim\limits_{t \to \infty} (t+3)(2t^2-1)^{-\frac{1}{2}} = \lim\limits_{t \to \infty} \sqrt{\dfrac{(t+3)^2}{2t^2-1}}$

$= \sqrt{\lim\limits_{t \to \infty} \dfrac{t^2+6t+9}{2t^2-1}} = \sqrt{\lim\limits_{t \to \infty} \dfrac{\frac{t^2}{t^2}+\frac{6t}{t^2}+\frac{9}{t^2}}{\frac{2t^2}{t^2}-\frac{1}{t^2}}}$

$= \sqrt{\lim\limits_{t \to \infty} \dfrac{1+\frac{6}{t}+\frac{9}{t^2}}{2-\frac{1}{t^2}}}$ as $t \to \infty$, $\frac{6}{t}=0$, $\frac{9}{t^2}=0$, $\frac{1}{t^2}=0$

$= \sqrt{\lim \dfrac{1+0+0}{2-0}} = \sqrt{\dfrac{1}{2}}$

$\lim\limits_{t \to \infty} \dfrac{t+3}{\sqrt{2t^2-1}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

4b.4e) $\lim\limits_{x \to \infty} \dfrac{-2}{3x+7}$

$$\lim_{x \to \infty} \frac{-2}{3x+7} = -2 \lim_{x \to \infty} \frac{1}{3x+7}$$

$$= -2 \left[\frac{1}{\lim\limits_{x \to \infty} (3x+7)} \right]$$

$$= -2 \left[\frac{1}{\lim\limits_{x \to \infty} 3x + \lim\limits_{x \to \infty} 7} \right]$$

$$= \frac{-2}{3 \lim\limits_{x \to \infty} x}$$

$$\lim_{x \to \infty} x = \infty$$

$$\frac{-2}{3 \lim\limits_{x \to \infty} x} = \frac{-2}{3 \cdot \infty} = 0$$

Scanned with CamScanner

Qn 5 Find y' and y''

5 a) $y = \cos(\sin\theta)$

Using the rule $\left[\cos(u(x))\right]' = -\sin(u(x)) \cdot u'(x)$

$$y' = \frac{d}{d\theta}\left[\cos(\sin\theta)\right)$$

$$= \left(-\sin(\sin\theta)\right) \cdot \frac{d}{d\theta}(\sin\theta)$$

$$= -\cos\theta \sin(\sin\theta)$$

$$y'' = \frac{d}{d\theta}\left[-\cos\theta\sin(\sin\theta)\right]$$

$$= \frac{-d}{d\theta}\left[\cos\theta\sin(\sin\theta)\right]$$

$$= -\left(\frac{d}{d\theta}\left[\cos(\theta)\right]\cdot\sin(\sin\theta) + \cos(\theta)\cdot\frac{d}{d\theta}\left[\sin(\sin\theta)\right]\right.$$

$$= -\left(-\sin\theta\sin(\sin\theta) - \cos(\theta)\cos(\sin\theta)\cdot\frac{d}{d\theta}(\sin\theta)\right)$$

$$= \sin\theta\sin(\sin\theta) - \cos^2(\theta)\cos(\theta)\cos(\sin\theta)$$

$$= \sin(\theta)\sin(\sin\theta) - \cos^2(\theta)\cos(\sin\theta)$$

5 b) $y = (1+\sqrt{x})^3$

$\frac{d}{dx}\left(y = (1+\sqrt{x})^3\right)$

$y' = \frac{d}{dx}(1+\sqrt{x})^3 = \frac{d}{dx}(\sqrt{x}+1)^3$

$= 3(\sqrt{x}+1)^2 \cdot \frac{d}{dx}(\sqrt{x}+1)$

$= 3(\sqrt{x}+1)^2\left(\frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}(1)\right)$

$= 3(\sqrt{x}+1)^2\left(\frac{1}{2}x^{1/2-1} + 0\right)$

$= \frac{3(\sqrt{x}+1)^2}{2\sqrt{x}}$

$y'' = \frac{d}{dx}\left[\frac{3(\sqrt{x}+1)^2}{2\sqrt{x}}\right)$

$= \frac{3}{2} \cdot \frac{d}{dx}\left[\frac{(\sqrt{x}+1)^2}{\sqrt{x}}\right)$

$\frac{3 \cdot \frac{\frac{d}{dx}\left[(\sqrt{x}+1)^2\right] \cdot \sqrt{x} - (\sqrt{x}+1)^2 \cdot \frac{d}{dx}\sqrt{x}}{(\sqrt{x})^2}}{2}$

$= \frac{3\left(2(\sqrt{x}+1) \cdot \frac{d}{dx}[\sqrt{x}+1] \cdot \sqrt{x} - \frac{1}{2}x^{1/2-1}(\sqrt{x}+1)^2\right)}{2x}$

$= \frac{3\left(2(\sqrt{x}+1)\left(\frac{d}{dx}[\sqrt{x}] + \frac{d}{dx}(1)\right)\sqrt{x} - \frac{(\sqrt{x}+1)^2}{2\sqrt{x}}\right)}{2x}$

$= \frac{3\left(2(\sqrt{x}+1)\left(\frac{1}{2}x^{\frac{1}{2}-1} + 0\right)\sqrt{x} - \frac{(\sqrt{x}+1)^2}{2\sqrt{x}}\right)}{2x}$

$= \frac{3\left(\sqrt{x} - \frac{(\sqrt{x}+1)^2}{2\sqrt{x}} + 1\right)}{2x}$

5 c) $y = \sqrt{\cos x}$

$$y' = \frac{d}{dx}\sqrt{\cos x}$$

$$= \frac{d}{dx}\cos^{\frac{1}{2}}(x)$$

$$= \frac{1}{2}\cos^{\frac{1}{2}-1}(x) \cdot \frac{d}{dx}\cos(x)$$

$$= \frac{-\sin x}{2\sqrt{\cos x}}$$

$$= -\left(\frac{\sin x}{2\sqrt{\cos x}}\right)$$

$$y'' = \frac{d}{dx}\left(-\frac{\sin x}{2\sqrt{\cos x}}\right) = \frac{-1}{2}\cdot\frac{d}{dx}\left(\frac{\sin x}{\sqrt{\cos x}}\right)$$

$$= -\frac{\dfrac{\frac{d}{dx}(\sin x)\cdot\sqrt{\cos x} - \sin(x)\frac{d}{dx}\left(\sqrt{\cos(x)}\right)}{\left(\sqrt{\cos(x)}\right)^2}}{2}$$

$$= -\frac{\cos(x)\sqrt{\cos(x)} - \frac{1}{2}\cos^{\frac{1}{2}-1}(x)\cdot\frac{d}{dx}\left(\cos(x)\right)\cdot\sin x}{2\cos(x)}$$

$$= -\frac{\cos^{\frac{3}{2}}(x) - \dfrac{(-\sin(x)\sin(x))}{2\sqrt{\cos(x)}}}{2\cos(x)}$$

$$= \frac{\dfrac{\sin^2(x)}{2\sqrt{\cos(x)}} + \cos^{3/2}(x)}{2\cos(x)}$$

$$= \frac{\sin^2(x)}{4\cos^{3/2}(x)} - \frac{\sqrt{\cos(x)}}{2}$$

$$= \frac{-\sin^2(x) + 2\cos^2(x)}{4\cos^{3/2}(x)}$$

5 d) $y = e^{e^x}$

$y' = \frac{d}{dx}\left(e^{e^x}\right)$

$= e^{e^x} \cdot \frac{d}{dx}\left(e^x\right)$

$= e^{e^x} \cdot e^x$

$= e^{e^x + x}$

$y'' = \frac{d}{dx}\left[e^{e^x + x}\right]$

$= e^{e^x + x} \cdot \frac{d}{dx}\left[e^x + x\right]$

$= e^{e^x + x}\left(\frac{d}{dx}\left[e^x\right] + \frac{d}{dx}\left[x\right]\right)$

$= e^{ex + x}\left(e^x + 1\right)$

Q^n 6 Find $\frac{dy}{dx}$ by Implicit Differentiation

6a) $x^2 - 4xy + y^2 = 4$

$$\frac{d}{dx}\left(x^2 - 4xy + y^2\right) = \frac{d}{dx}(4)$$

L.H.S Treat y as a function of x

$$\frac{d}{dx}\left(x^2 - 4xy(x) + y^2(x)\right) = \frac{d}{dx}(x^2) - \frac{d}{dx}(4xy(x)) + \frac{d}{dx}(y^2(x))$$

$$= -2\left(2x \frac{d}{dx} y(x) - x - y(x)\frac{d}{dx}(y(x)) + 2y(x)\right)$$

RHS

$$\frac{d}{dx}(4) = 0$$

$$\Rightarrow -4x\frac{dy}{dx} + 2x + 2y\frac{dy}{dx} - 4y = 0$$

$$\frac{dy}{dx} = \frac{x - 2y}{2x - y}$$

6b) $2x^2 + xy - y^2 = 2$

$$\frac{d}{dx}(2x^2 + xy - y^2) = \frac{d}{dx}(2)$$

LHS Treat y as a function of x

$$\frac{d}{dx}(2x^2 + xy(x) - y^2(x)) = \frac{d}{dx}(2x^2) + \frac{d}{dx}(xy(x)) - \frac{d}{dx}(y^2(x))$$

$$= x\frac{d}{dx}(y(x)) + 4x - 2y(x)\frac{d}{dx}(y(x)) + y(x)$$

RHS $\frac{d}{dx}(2) = 0$

$$\Rightarrow x\frac{dy}{dx} + 4x - 2y\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\left(\frac{4x+y}{x-2y}\right)$$

6c) $x^4 + x^2y^2 + y^3 = 5$

$$\frac{d}{dx}(x^4 + x^2y^2 + y^3) = \frac{d}{dx}(5)$$

LHS Treat y as a function of x.

$$\frac{d}{dx}(x^4 + x^2y^2 + y^3) = \frac{d}{dx}(x^4) + \frac{d}{dx}(x^2y^2(x)) + \frac{d}{dx}(y^3(x))$$

$$= 4x^3 + 2x^2y(x)\frac{d}{dx}y(x) + 2xy^2(x) + 3y^2(x)\frac{d}{dx}(y(x))$$

RHS $\frac{d}{dx}(5) = 0$

$$\Rightarrow 4x^3 + 2x^2y\frac{dy}{dx} + 2xy^2 + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\left(\frac{2x(2x^2+y^2)}{y(2x^2+3y)}\right)$$

6d) $x^3 - xy^2 + y^3 = 1$

$\frac{d}{dx}(x^3 - xy^2 + y^3) = \frac{d}{dx}(1)$

LHS Treat y as a function of x

$\frac{d}{dx}(x^3 - xy^2 + y^3) = \frac{d}{dx}(x^3) - \frac{d}{dx}(xy^2(x)) + \frac{d}{dx}(y^3(x))$

$= 3x^2 - 2xy(x)\frac{d}{dx}(y(x)) + 3y^2(x)\frac{d}{dx}y(x) - y^2(x).$

RHS $\frac{d}{dx}(1) = 0$

$\Rightarrow 3x^2 - 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} - y^2 = 0$

$$\frac{dy}{dx} = \frac{3x^2 - y^2}{y(2x - 3y)}$$